On Primary User Detection Using Energy Detection Technique For Cognitive Radio

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***Abstract*— In this paper we have examined spectrum sensing using energy detection technique for cognitive radio. We have calculated the probability of detection, probability of false alarm and the probability of error in detecting primary users for complex Gaussian signal and evaluated the effect of different sensing parameters on the probability of error in detecting primary users.**

I. Introduction

Intelligent channel sensing forms the basic technique towards the evolution of cognitive radio technology. Cognitive radios work on efficient sensing algorithms to find the white spaces in the primary/licensed user band and utilize them to transmit their own data [1].

The key performance parameters for sensing algorithms are the probability of detection, P_D, implying that a primary user is correctly sensed to be present when it is actually present and the probability of false alarm, P_F, implying that a primary user is falsely signalled to be present when it is actually not present.

In [2], authors have calculated the probability of detection and probability of false alarm using FFT techniques for a typical case of OFDM subcarriers. In [3], authors have calculated probability of detection and probability of false alarm for PSK signals and have calculated the tradeoff between sensing time and throughput.

Figure 1shows a typical binary hypothesis testing scenario. H_0 and H_1 are the two hypothesis used which denote the absence and the presence of the primary user respectively. The quality of detection can be measured by the following parameters

- P_D as the probability of detection.
- P_F as the probability of false alarm.
- P_M as the probability of miss and is given by $1 - P_D$.

The threshold parameter γ is used in conducting likelihood ratio tests to determine the hypothesis H_0 or H_1.

It is desirable to have a higher value of P_D and a lower value of P_F. This is because higher P_D implies primary users are better protected and lower P_F implies more chances of secondary users to reuse the channel which in turn will lead to higher throughput. Thus in cognitive radio networks for increasing secondary user throughput, the problem is of minimizing P_F and increasing P_D.

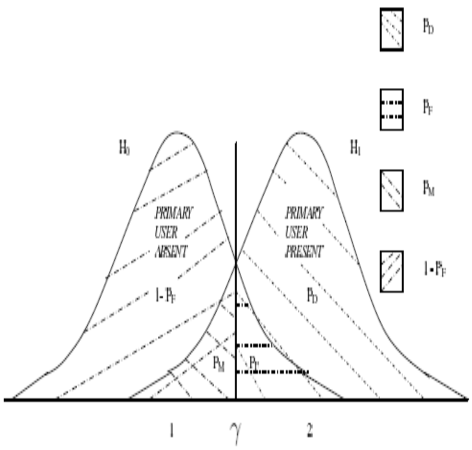

Fig. 1. Binary Hypothesis Testing

However, reducing P_F and increasing P_D are conflicting objectives. Thus there is need of a tradeoff in the achievable values of the two probabilities.

Basically, the main objective to work with conditional probabilities such as P_F and P_D is to bypass the difficulties faced in assigning realistic costs or a priori probabilities. By realistic costs we mean the function that would denote the real time cost function considering the presence and the absence of the primary user. To generate real time cost function in varied fading and interference scenarios would require very strong a priori knowledge of parameters such as channel gain, bit error rate etc. This sort of precondition would render any system practically infeasible.

This paper is organized as follows. In section II, we have derived the expression for probability of detection, P_D, and probability of false alarm, P_F of primary user under energy sensing technique, and observed the effect of variation in the threshold γ and number of samples, N on quality of sensing. In section III, we have calculated the probability of error in detecting primary user, and observed its relation to variation in probability of false alarm, number of samples, primary user occupancy, and signal-to-noise ratio (SNR). Finally, we conclude the paper by summarizing the main results of the paper in section IV.

II. Probability of Detection and Probability of False Alarm of Primary User under Energy Sensing Technique

The binary hypothesis H_0 and H_1 are defined as
H_0 : (primary user absent)

$$y(n) = u(n) \qquad n = 1, 2, \ldots, N \tag{1}$$

H_1 : (primary user present)

$$y(n) = s(n) + u(n) \qquad n = 1, 2, \ldots, N \tag{2}$$

We make the following assumptions [2] :

- $u(n)$ is AWGN noise defined as $CN(0, \sigma_u^2)$.
- $s(n)$ is the primary user signal defined as $CN(0, \sigma_s^2)$.
- $s(n)$ and $u(n)$ are independent.
- Primary user signal to noise ratio under H_1 is denoted by

$$\alpha = \frac{\sigma_s^2}{\sigma_u^2} \tag{3}$$

Energy detection technique for channel sensing is best suited for low complexity receivers in high signal to noise ratios. The test statistic for energy detector is given as [4]

$$T(y) = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^2 \tag{4}$$

where N denotes the number of samples and $|y(n)|^2$ is chi-squared distributed [5].

Now, under H_0, $T(y)$ is a random variable with PDF $p_0(x)$ (chi-squared distributed) and $2N$ degrees of freedom for complex $y(n)$ and N degrees of freedom for real $y(n)$.

For a chosen threshold level γ, we have,

$$P_F = P[T(y) \geq \gamma / H_0] = \int_\gamma^\infty p_0(x)dx \tag{5}$$

Under H_0

$$y(n) = u(n),$$

$$T(y) = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^2 = \frac{1}{N}\sum_{n=1}^{N} |u(n)|^2 \tag{6}$$

Mean, denoted as μ_0 is given by [5],

$$\mu_0 = E[T(y)] = \frac{1}{N}\sum_{n=1}^{N} \sigma_u^2 = \sigma_u^2 \tag{7}$$

Variance, denoted as σ_0^2 is given by [6]

$$\sigma_0^2 = E[T(y) - \mu_0]^2 = \frac{1}{N}[E|u(n)|^4 - \sigma_u^4] = \frac{1}{N}\sigma_u^4 \tag{8}$$

Using central limit theorem for large N, $p_0(x)$ can be approximated as Gaussian PDF [5].

$$\begin{aligned} P_F &= \int_\gamma^\infty N(\mu_0, \sigma_0^2)dx \\ &= \frac{1}{\sigma_0\sqrt{2\pi}}\int_\gamma^\infty e^{-\frac{(x-\mu_0)^2}{2\sigma_0^2}}dx \end{aligned} \tag{9}$$

Using change of variables, we can write eq. 9 as

$$\begin{aligned} P_F(\gamma) &= \frac{1}{\sqrt{2\pi}}\int_{[(\frac{\gamma}{\sigma_u^2}-1)\sqrt{N}]}^\infty e^{\frac{-t^2}{2}}dt \\ &= Q\left[\left(\frac{\gamma}{\sigma_u^2} - 1\right)\sqrt{N}\right] \end{aligned} \tag{10}$$

where, $Q(.)$ is defined as

$$Q(\gamma) = \frac{1}{\sqrt{2\pi}}\int_\gamma^\infty e^{-\frac{t^2}{2}}dt \tag{11}$$

Under H_1,

$$y(n) = s(n) + u(n),$$

Denoting the PDF of $T(y)$ under H_1 as $p_1(x)$ we get P_D as

$$P_D = P[T(y) \geq \gamma / H_1] = \int_\gamma^\infty p_1(x)dx \tag{12}$$

Here again we will use central limit theorem for large N, such that $p_1(x)$ can be approximated as Gaussian PDF.

Mean, denoted as μ_1 is given by

$$\mu_1 = E[T(y)] = \left[\frac{\sigma_s^2}{\sigma_u^2} + 1\right]\sigma_u^2 = [\alpha + 1]\sigma_u^2 \tag{13}$$

Variance, denoted as σ_1^2 is given by

$$\sigma_1^2 = E[T(y) - \mu_1]^2 = \frac{1}{N}[\alpha + 1]^2\sigma_u^4 \tag{14}$$

Using eq. 13 and 14 for mean and variance, we get

$$\begin{aligned} P_D &= \int_\gamma^\infty N(\mu_1, \sigma_1^2)dx \\ &= \frac{1}{\sigma_1\sqrt{2\pi}}\int_\gamma^\infty e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}dx \end{aligned} \tag{15}$$

Similarly, using change of variables, we can write eq. 15 as

$$\begin{aligned} P_D(\gamma) &= \frac{1}{\sqrt{2\pi}}\int_{\frac{\sqrt{N}}{\alpha+1}(\frac{\gamma}{\sigma_u^2}-\alpha-1)}^\infty e^{\frac{-t^2}{2}}dt \\ &= Q\left[\frac{\sqrt{N}}{\alpha+1}\left(\frac{\gamma}{\sigma_u^2} - \alpha - 1\right)\right] \end{aligned} \tag{16}$$

Using eq. 10 and 16, we obtain an interdependent system of equations as

$$P_F = Q[(\alpha + 1)Q^{-1}(P_D) + \sqrt{N}\alpha] \tag{17}$$

$$P_D = Q\left[\frac{1}{\alpha+1}Q^{-1}(P_F) + \frac{\sqrt{N}\alpha}{\alpha+1}\right] \tag{18}$$

Now we observe the effect of threshold γ and number of samples on P_D and P_F.

- **Variation in threshold,** γ**:** As γ increases, both P_D and P_F decreases, and vice-versa.
- **Variation in number of samples,** N**:** For $\frac{\gamma}{\sigma_u^2} > 1$, P_F decreases with increase in N, and vice-versa, while for $\frac{\gamma}{\sigma_u^2} > \alpha + 1$, P_D decreases with increase in N, and vice-versa.

III. Calculation of Probability of Error in Detecting Primary Users

Let the probability of primary user occurrence be given as P. Thus the probability of non-occurrence is given by $1-P$. Then the probability of detecting error is given by

$$P_r(e) = (1-P)P_F + P(1-P_D) \quad (19)$$

From eq. 19 it can be seen that maximizing P_D and minimizing P_F will reduce $P_r(e)$. Probability of detecting error can be written as

$$P_r(e) = (1-P)P_F + P\left(1-Q\left[\frac{\sqrt{N}}{\alpha+1}\left(\frac{\gamma}{\sigma_u^2}-\alpha-1\right)\right]\right) \quad (20)$$

If the preset value of P_F is $\hat{\beta}$, then

$$P_r(e) = (1-P)\hat{\beta} + P\left(1-Q\left[\frac{\sqrt{N}}{\alpha+1}\left(\frac{\gamma}{\sigma_u^2}-\alpha-1\right)\right]\right) \quad (21)$$

Thus, $P_r(e)$ is a function of P, $\hat{\beta}$, N, α and γ. Mathematically,

$$P_r(e) = f(P, \hat{\beta}, N, \alpha, \frac{\gamma}{\sigma_u^2}) \quad (22)$$

We will now evaluate the effect of different parameters on the probability of detecting error, $P_r(e)$.

- **Variation in probability of false alarm, β:**
  We first considered the case when the threshold $\frac{\gamma}{\sigma_u^2} < \alpha+1$ (see Figure 2). The value of $P_r(e)$ varies as the slope $1-P$. When P_F is less, the second term in eq. 19 dominates, thus for higher values of P we get higher values of $P_r(e)$. When P_F is high, then the first term in eq. 19 dominates, so for decreasing P, we get higher value of $P_r(e)$. Thus to reap benefits of very low probability of occurrence of primary users, as $P = 0.1$, we need to have lower value of P_F as for higher P_F even the lower P regions will show high value of $P_r(e)$. For black spaces with P typically being 0.9 or higher, $P_r(e)$ variation is very small with P_F. This is as expected as the second term dominates in eq. 19 for larger value of P.
  For the case, when the threshold $\frac{\gamma}{\sigma_u^2} > \alpha+1$ (see Figure 3), when P_F is less, then $P_r(e) \sim P$. Thus increasing P increases $P_r(e)$. Again, when P_F is high, then $P_r(e) \sim (1-P)P_F + P$, and thus increasing P increases $P_r(e)$. In this case, even if we increase P_F, we still get lower value of $P_r(e)$ for lower P unlike that as obtained in Figure 2.
- **Variation in number of samples, N:** In Figure 4, probability of detecting error is plotted vs. the variation in number of samples. When the ratio $\frac{\gamma}{\sigma^2} < \alpha+1$, for large values of N, $P_r(e) \approx (1-P)P_F$. Now P and P_F being constant for this particular setting, we get a constant value of $P_r(e)$ for large N. Similar observation is made for the case when $\frac{\gamma}{\sigma^2} > \alpha+1$. Thus we can see that whatever be the number of samples, increased value of P always gives higher value of $P_r(e)$ for a particular N.
- **Variation in primary user occupancy, P:**
  In Figure 5 the variation in $P_r(e)$ for different probability of primary user occurrence P is shown for $\frac{\gamma}{\sigma_u^2} < \alpha+1$ is shown. For higher P_F, the slope is negative and for lower value of P_F, slope is positive, as evident from the equation 19. For lower P, the first term of equation 19 dominates, and thus higher value of P_F gives higher value of $P_r(e)$. Now when $P = 1$, $P_r(e)$ is independent of P_F, So all the lines converge to a single point independent of the value of P_F.
  In Figure 6 the variation in $P_r(e)$ for different probability of primary user occurrence P is shown for $\frac{\gamma}{\sigma_u^2} > \alpha+1$ is shown. For large values of N and $\frac{\gamma}{\sigma_u^2} > \alpha+1$, the slope is approximately given by $1-P_F$. Hence, unlike the case of $\frac{\gamma}{\sigma_u^2} < \alpha+1$, here the slope is always positive.
- **Variation in SNR, α:**
  In Figure 7, the variation in $P_r(e)$ is plotted as a function of the SNR α for the case when $\frac{\gamma}{\sigma_u^2} < \alpha+1$. When probability of false alarm P_F is small, the second term

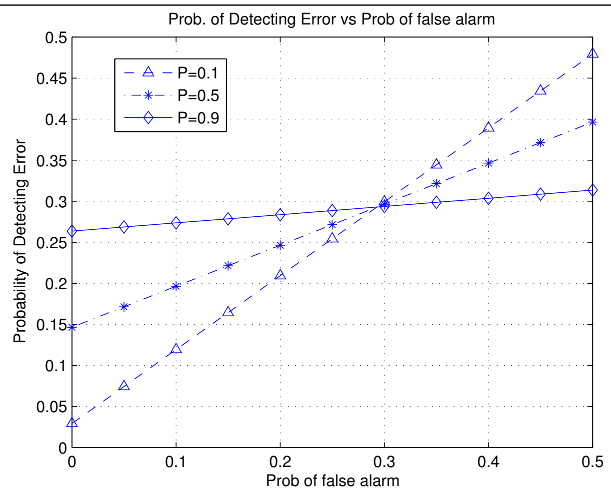

Fig. 2. Probability of detecting error v/s probability of false alarm for $\gamma < 11$, SNR = 10 dB, N=1000, γ= 10.

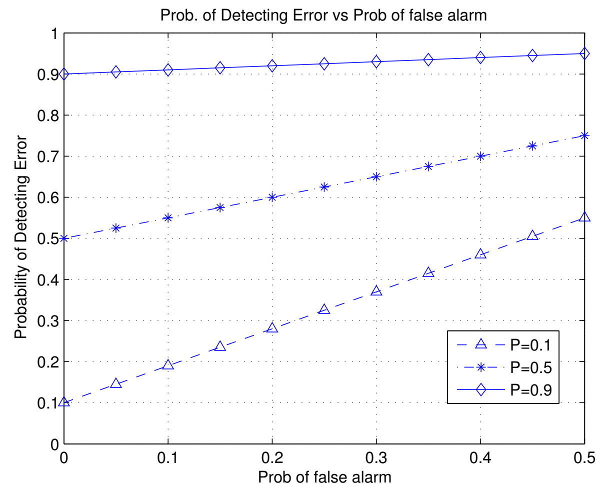

Fig. 3. Probability of detecting error v/s probability of false alarm for $\gamma > 11$ SNR = 10 dB, N=1000, γ= 12.

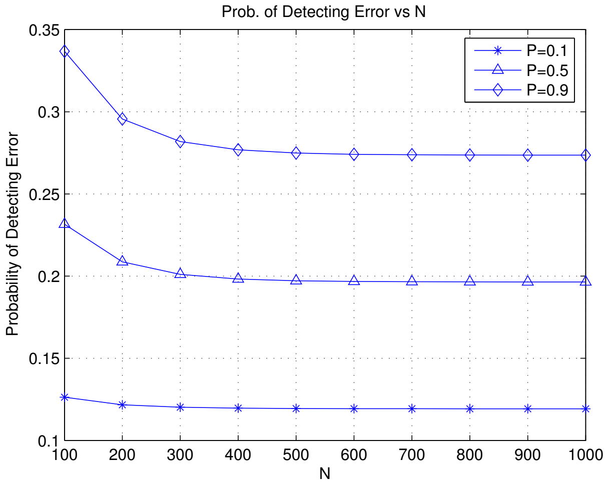

Fig. 4. Prob. of Detecting Error vs N for $\gamma < 11$, SNR = 10 dB, P_F=0.1, γ= 10.

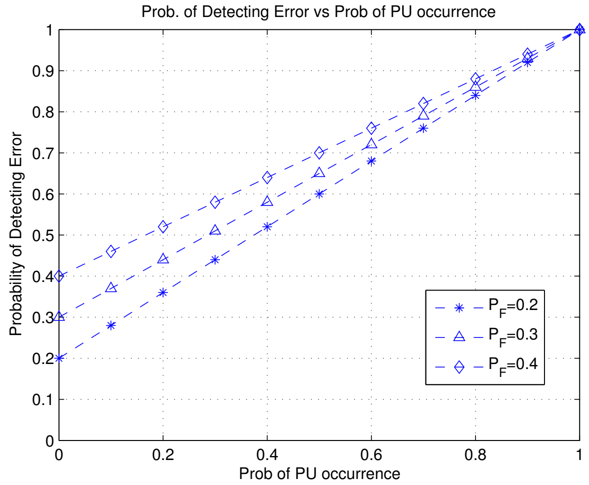

Fig. 6. Prob. of Detecting Error vs Prob of PU occurrence for $\gamma > 11$ SNR = 10 dB, N=1000, γ= 12.

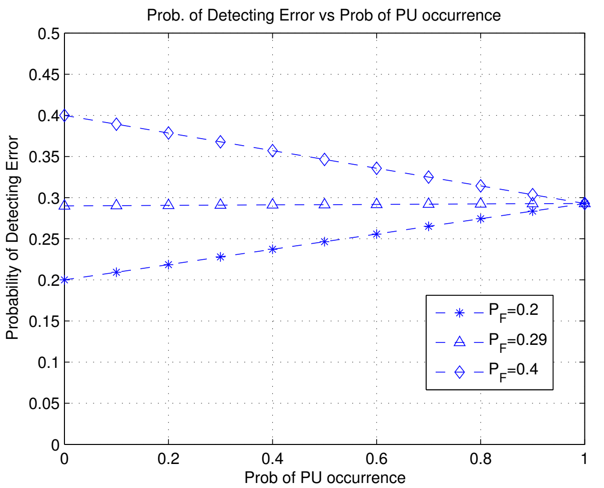

Fig. 5. Prob. of Detecting Error vs Prob of PU occurrence for $\gamma < 11$ SNR = 10 dB, N=1000, γ= 10.

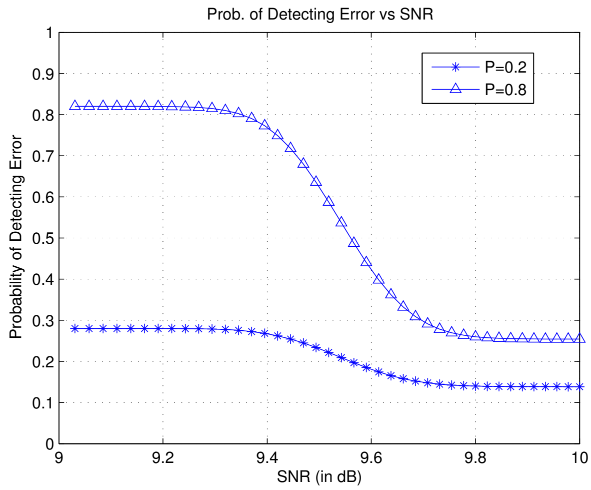

Fig. 7. Prob. of Detecting Error vs SNR (in dB) for $\gamma < 11$, N=1000, P_F=0.1, γ= 10.

in eq. 19 dominates, thus for higher value of P, we get a correspondingly higher value of $P_r(e)$. Also higher value of SNR results in lower $P_r(e)$. For the case when $\frac{\gamma}{\sigma_u^2} > \alpha + 1, P_r(e) \approx (1 - P)P_F$, thus almost constant with respect to change in SNR.

IV. Conclusions

In this paper we have derived the expression for the probability of detection and the probability of false alarm using the binary hypothesis testing procedure for spectrum sensing of the primary user using energy detection technique. We have also calculated the probability of error in detecting the primary users and observed the effect of different sensing parameters on the quality of sensing.

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