Low Density Parity Check codes in OFDM system

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***Abstract*— In this work, we study the performance of Low Density Parity Check (LDPC) codes over an Orthogonal Frequency Division Multiplexing (OFDM) channel. We state a concentration theorem which shows that no Gaussian approximation is required in the analysis of LDPC codes over OFDM. Then we propose a rigorous density evolution method (without Gaussian approximations) to prove the existence of thresholds for LDPC codes over OFDM and evaluate the thresholds for various regular and irregular LDPC codes. We calculate the capacity of OFDM channel and compare LDPC threshold with this theoretical limit and show that for irregular codes, LDPC thresholds are very close to capacity at higher rates. We also compare the LDPC threshold in OFDM with LDPC threshold in an ISI channel with BCJR equalization. Using the feedback to the transmitter, we apply Mercury/Waterfilling power allocation to improve the OFDM capacity and LDPC thresholds. We show that with Mercury/Waterfilling power allocation LDPC thresholds are very close to capacity even at moderate rates .**

I. Introduction

Low Density Parity Check (LDPC) codes exhibit a threshold phenomenon when iteratively decoded using a sum-product message passing decoder over many channels. Arbitrarily low bit-error rates (BERs) can be obtained whenever the channel noise level is below a particular threshold value by increasing the blocklength and number of iterations. An algorithm for finding the threshold, Density Evolution (DE), has been proposed by Richardson *et al* [1]. For channels with additive Gaussian noise, threshold DE was simplified by Chung *et al* [2] using a Gaussian approximation. DE has been extended to binary-input inter-symbol interference (ISI) channels by Kavcic *et al* [3], who show that LDPC codes provide near capacity performance over discrete-time Inter Symbol Interference (ISI) channels.

The general problem of channel coding for OFDM systems has been addressed in [4]. Design optimization of LDPC codes for Multiple Input Multiple Output-OFDM (MIMO-OFDM) system for a fixed target data rate has been addressed in [5]. Mannoni *et al* proposed a linear criterion for the optimization of irregular LDPC codes for an OFDM system [6]. Baynast *et al* [7] have proposed a two-step optimization of irregular LDPC codes for OFDM channels. All of these previous works employ a Gaussian approximation for threshold estimation and do not completely prove the existence of thresholds. In [11] we proposed a density evolution algorithm without Gaussian approximation and presented initial results on thresholds calculated for LDPC codes over OFDM. In this paper, we address the following: (1) Proof of the concentration theorem, (2) Mercury/Waterfilling power allocation across the OFDM subcarriers, (3) More accurate computation of thresholds for irregular and regular LDPC codes, (4) Comparison with OFDM, ISI capacities.

Specifically, we state a concentration theorem which shows that no Gaussian assumption is necessary in the analysis of LDPC codes over OFDM. Using this result we then propose a rigorous density evolution algorithm to compute threshold for LDPC codes over an ISI channel under OFDM. We assume that one code block is transmitted using a single OFDM symbol. In the algorithm, we allow the block length to tend to infinity. Consequently, the subcarrier spacing reduces and the number of subcarriers tend to infinity for the same bandwidth. Since the number of subcarriers tend to infinity, the finite cyclic prefix results in no additional overhead. We calculate the OFDM channel capacity and compare OFDM thresholds obtained by our density evolution with this theoretical limit. We show that for higher rates (rates higher than 0.6) the thresholds are very close to the theoretical limit. An optimum power allocation scheme, Mercury/Waterfilling, for parallel Gaussian channel with arbitrary input constellation has been proposed by Lozano *et al* [10]. We use this power allocation scheme to improve the OFDM capacity. We apply LDPC codes with this power allocation and demonstrate that LDPC thresholds also show considerable improvement. We show that, with this optimum power allocation, LDPC thresholds are very close to capacity even at moderate rates (rates higher than 0.2). We also make a comparison between the time-domain BCJR algorithm and the frequency-domain OFDM method for equalizing an ISI channel. To this end, we compare the threshold for LDPC codes under OFDM with that of the BCJR thresholds obtained by using the algorithm given in [3].

II. System Description

In this work, we focus our attention on an *Orthogonal Frequency Division Multiplexing* (OFDM) system. We assume transmission over an ISI channel[1] with L fixed taps. The channel is modeled as

$$\mathbf{z} = \mathbf{H}.\mathbf{c} + \mathbf{N}, \tag{1}$$

where $\mathbf{c}$ is the input vector, $\mathbf{z}$ the output vector, $\mathbf{H}$ the Discrete Fourier Transform (DFT) of the *Channel Impulse Response* and $\mathbf{N}$ the normalized DFT of the random noise vector, and "." denotes the dot product of two vectors. All these vectors mentioned here are of length N_c, the number of

[1]This can be easily extended to OFDM over a block fading channel.

subcarriers. We have input alphabet $\mathcal{X} = \mathbb{F}_2$ and consider BPSK modulated input $0 \to +1, 1 \to -1$. Therefore, the power in every input symbol is the same and equal to unity.

The analysis of the decoder requires letting the blocklength of the code to tend to infinity. The motive behind the assumption is that the cyclic prefix involved in the OFDM transmission remains an overhead of fixed length (given by the number of taps in the channel), while the number of information symbols increases as the OFDM symbol length tends to infinity. This increases the throughput, and in the limit the cyclic prefix gives no overhead. The above assumption would also imply that the number of subcarriers N_c would tend to infinity. In the OFDM system, the length of all vectors would also tend to infinity and the Inverse Discrete Fourier Transform (IDFT) and DFT computed would also be *infinite-point* versions. In the mathematical model for the channel, the factor multiplying the signal is now a sample of the Discrete Time Fourier Transform (DTFT) of the channel impulse response instead of a DFT sample in the finite version.

III. Analysis of LDPC codes over AWGN

For irregular LDPC codes of blocklength n, the parameters, the variable and check node degree distributions are specified as polynomials, denoted $\lambda(x)$ and $\rho(x)$, respectively. The triplet, (λ, ρ, n), thus specifies an ensemble of LDPC codes.

A. Density Evolution

Density Evolution is an algorithm that analyzes an ensemble of LDPC codes by tracking the probability density function (pdf) of the message passed on a random edge of a graph in the ensemble averaged over the entire ensemble. It can therefore specify the average error probability as a function of iteration number, thereby serving as a performance metric for LDPC codes.

Consider an AWGN channel, with binary input and BPSK modulation. The actual message passed on the edges in case of AWGN channel in the zeroth iteration of message passing is the *Log-Likelihood Ratio* (LLR) of the received value from the channel. The pdf of the LLR is termed the *L-density* and the L-density at iteration l is denoted f_l. It is assumed that there are no cycles up to depth l. Density evolution, in this case, states

$$\begin{aligned} l(y) &\sim f_0 \equiv \mathcal{N}\left(\frac{2}{\sigma^2}, \frac{4}{\sigma^2}\right) \\ f_l &= f_0 \otimes \lambda\left(\rho\left(f_{l-1}\right)\right), \end{aligned} \tag{2}$$

where for L-density f

$$\lambda(f) := \sum_i \lambda_i f^{\otimes(i-1)}, \qquad \rho(f) := \sum_i \rho_i f^{\boxtimes(i-1)}.$$

Here, $\otimes$ and $\boxtimes$ denote the convolutions carried out in the L-domain and G-domain, respectively. These domains are defined in [9].

The probability of error obtained is a monotone function with respect to the channel parameter (noise variance σ^2) and with respect to iteration number. Also, there exists a well-defined supremum of σ for which *probability of error* $\to 0$ as the number of iterations $l \to \infty$, and this supremum is called the *threshold* of the decoder, denoted σ^* [9].

IV. Analysis of LDPC codes over OFDM

A. Log-Likelihood Ratio

The OFDM channel described is clearly a binary memoryless channel. The LLR defined as

$$u_i = L(z_i) := \ln\left[\frac{p_{Z_i|C_i}(z_i|c_i = 1)}{p_{Z_i|C_i}(z_i|c_i = -1)}\right]$$

forms a sufficient statistic with respect to decoding for all binary memoryless channels.

B. Channel Symmetry

We see that

$$p_{Z_i|C_i}(z_i|c_i = 1) = p_{Z_i|C_i}(-z_i|c_i = -1)$$

for z_i given by (1). Thus, the channel is symmetric and can be modelled [9] as

$$c_i \xrightarrow{f_{U_i}} Z_i, \qquad Z_i = c_i U_i, \qquad U_i \sim f_{U_i} \tag{3}$$

where f_{U_i} is the distribution of u_i conditioned on $c_i = 1$ and channel gain $H[i]$:

$$f_{U_i}(u_i) = \frac{\sigma}{4|H[i]|\sqrt{\pi}} \exp\left[-\frac{(\sigma^2 u_i - 4|H[i]|^2)^2}{16|H[i]|^2\sigma^2}\right] \tag{4}$$

i.e. $U_i \sim \mathcal{N}\left(\frac{4|H[i]|^2}{\sigma^2}, \frac{8|H[i]|^2}{\sigma^2}\right)$. We see that

$$f_{U_i}(u_i) = \exp(u_i).f_{U_i}(-u_i) \tag{5}$$

and thus, the LLR distribution is symmetric. Under these symmetry conditions, we can assume that the transmitted code word is all-one codeword[2].

C. Concentration Theorem

In an OFDM channel, message passing decoding with an irregular LDPC code raises an interesting question. OFDM system can be considered as a set N parallel AWGN channel, each with a different SNR. For irregular codes, the degree of each bit node can be different. The incoming bit from the ith channel can be assigned to jth bit node for decoding. Since we are assuming that the length of OFDM symbol is same as the code length, there are $N!$ such assignments possible (all of them need not be different since there are many number of bit nodes with same degree). For example, the incoming bit from the highest SNR channel can be assigned to bit node with lowest degree and the incoming bit from the lowest SNR channel can be assigned to bit node with highest degree. All the other bits follows this order. We can think of another arrangement in the opposite order too. So, the problem is to find out an assignment which gives the optimum performance

[2]BPSK modulated

in terms of BER and LDPC threshold. This is equivalent to the problem of designing an optimum interleaver.

However, in our study, we observed that BER performance (and LDPC threshold) of an irregular LDPC code is almost the same for different random interleaving (incoming bits are assigned to the bit nodes in a random manner). More specifically, out of the $N!$ interleavers available, if we select interleaver u and v uniformly at random, their performance turned out to be remarkably close. We present this result as a concentration theorem. The proof is given in [12]

Consider a degree distribution pair (λ, ρ) and transmission over an OFDM channel with N subcarriers. We assume that the block length of the LDPC code n is same as the number of OFDM subcarrier *ie*, $n = N$. We denote each of these interleaver (or,assignment) as vectors $\underline{G}_u$, $1 \leq u \leq N!$.

1) Theorem: Let $Z^l_{\underline{G}_u}$ be the random variable that denotes the number of erroneous variable-to-check node messages after l rounds of the message-passing decoding algorithm when the code graph is chosen uniformly at random from the ensemble of graphs with degree distribution pair (λ, ρ) and when the the interleaver chosen uniformly at random is $\underline{G}_u$. Let $p^l_{\underline{G}_u}$ be the expected number of incorrect messages along an edge with a tree-like neighborhood of depth atleast $2l$ at the lth iteration when the interleaver chosen uniformly at random. Let n_e be the number of edges in the graph. For an arbitrary small constant $\epsilon > 0$, there exists a positive constant $\beta = \beta$ (λ, ρ, l), such that if $n > \frac{2\gamma}{\epsilon}$, then

$$P\left(\left|\frac{Z^l_{\underline{G}_u}}{n_e} - \overline{p}\right| \geq \epsilon\right) \leq 4e^{-\beta' \epsilon^2 n} \tag{6}$$

where the *error concentration probability* $\overline{p}$ is defined as

$$\overline{p} = \frac{1}{N!}\sum_{i=1}^{N!} p^l_{\underline{G}_u}.$$

The theorem shows that $Z^l_{\underline{G}_u}$ is highly concentrated around $\overline{p}$. This result ensures that we need not consider any particular interleaver for the analysis of LDPC over OFDM since the performance given by an interleaver selected uniformly at random from the set of all arrangement is close to the average performance and hence it is enough to study this average behaviour. This eliminates the need for Gaussian approximation in the density evolution and enables us to propose a rigorous density evolution which analyze this average behavior.

D. Density Evolution

Theorem: Consider an OFDM channel with N_c subcarriers with code of blocklength $n = N_c$, with associated L-densities $\tilde{f}_i, i \in \{1, 2, \ldots, N_c\}$ (4). Then, the initial message density

$$f_0 = \frac{1}{N_c}\sum_{i=1}^{N_c} \tilde{f}_i, \tag{7}$$

and for $l \geq 1$,

$$f_l = f_0 \otimes \lambda\left(\rho\left(f_{l-1}\right)\right), \tag{8}$$

where for L-density f

$$\lambda(f) := \sum_i \lambda_i f^{\otimes(i-1)}, \qquad \rho(f) := \sum_i \rho_i f^{\boxtimes(i-1)}.$$

Proof: The initial density of the LLR f_0 is the only step of the algorithm that differs from the AWGN channel case. It can be easily seen that the f_0 given by (7) is still symmetric, i.e. it still satisfies (5). We now prove that the initial density of the LLR is given by (7). The proof of the rest of the algorithm is exactly the same as in the AWGN channel case and is given in [9].

Let e_r be a random edge and v_i the ith variable node in the Tanner graph G of the code specified by the degree distribution pair (λ, ρ). Let N_e be the total number of edges in G. The LLR distribution of the message received at v_i from the channel is given by $\tilde{f}_i$. The probability density function of the message carried by this edge in the variable-to-check message passing step of the zeroth iteration, averaged over the ensemble of graphs characterized by (λ, ρ) is given by

$$\begin{aligned} f_0 :=& \mathbb{E}_{[G(\lambda,\rho)]}(f(e_r)) \\ =& \sum_{i=1}^{N_c} \mathrm{P}(v_i \in e_r)\tilde{f}_i \\ =& \sum_{i=1}^{N_c}\left[\sum_m \mathrm{P}\left(d_G(v_i) = m\right).\mathrm{P}\left(v_i \in e_r | d_G(v_i) = m\right)\right]\tilde{f}_i \\ =& \sum_{i=1}^{N_c}\left[\sum_m \left\{\frac{N_e \lambda_m}{m N_c}\right\}\left\{\frac{m}{N_e}\right\}\right]\tilde{f}_i \\ =& \frac{1}{N_c}\sum_{i=1}^{N_c}\tilde{f}_i \end{aligned}$$

which is the same as (7).

E. Monotonicity and Threshold

As the update equation involved in the density evolution algorithm is the same as that in the AWGN case, the monotonicity arguments made there apply here also. We therefore have a supremum σ^* for the noise variance beyond which the error probability does not converge to zero even after infinite rounds of message passing.

We estimate these thresholds for various regular LDPC codes over OFDM channels. We describe the method employed for threshold estimation in the following section.

V. Threshold Estimation

Since we are assuming a single OFDM symbol over the blocklength, we let N_c tend to infinity. Equation (7) is no more a summation but an integral. As described earlier, the LLR distribution is now a continuous function of ω the angular frequency through its dependence on $H(e^{j\omega})$ the DTFT of the channel impulse response. Equation (4) therefore becomes

$$f(u,\omega) = \frac{\sigma}{4|H(e^{j\omega})|\sqrt{\pi}} \exp\left[-\frac{(\sigma^2 u - 4|H(e^{j\omega})|^2)^2}{16|H(e^{j\omega})|^2\sigma^2}\right]$$
$$H(e^{j\omega}) = \sum_{i=-\infty}^{\infty} h[i]e^{-j\omega_i}$$

and (7) becomes

$$f_0(u) = \frac{1}{2\pi}\int_0^{2\pi} f(u,\omega).d\omega \qquad (9)$$

Unfortunately, the function $f(u,\omega)$ is not always well behaved - it tends to the continuous Dirac-Delta function when $|H(e^{j\omega})| = 0$ and therefore $f_0(u)$ is not directly obtainable from (9) when the channel has spectral nulls. This difficulty can be overcome by calculating $f_0(u)$ through the characteristic function of $f(u,\omega)$ [11].

The density given by this method is now used in the density evolution to estimate the threshold.

VI. Optimum Power Allocation Using Mercury/Waterfilling

OFDM can be considered as a set on N_c parallel AWGN channel. On the ith channel, the input-output relation is

$$Y_i = H_i X_i + W_i, \qquad (10)$$

where the complex scalar H_i is the deterministic gain while the noise W_i is a zero mean unit variance complex Gaussian random variable independent of the noise of the other channel. The aggregate power constraint is $\frac{1}{N_c}\sum_{i=1}^{N_c} E\left[|X_i|^2\right] \leq P$. Once we know the expected value of the signal power in each channel, calculating the capacity of that channel is fairly straight forward. Since we can calculate the capcity for each parallel AWGN channel, the capacity of OFDM system can also be computed easily.

The SNR corresponds to each channel can be different due to different H_i which scales the signal. Therefore, the trivial power allocation, equal power on all subcarrier, will not be the optimum. If the input to the parallel channels are mutually independent and Gaussian, the optimum power allocation is simple and given by the waterfilling policy. However the inputs are usually drawn from a discrete constellation, and the waterfilling policy is no longer optimum. The main difficulty in the formulation is the lack of explicit expression for the corresponding mutual information. Recently, Mercury/Waterfilling scheme which is the optimum power allocation for parallel Gaussian channels with arbitrary input constellation has been proposed by Lozano *et al* [10].

We used the Mercury/Waterfilling power allocation scheme and calculated the OFDM system capacity. OFDM system capacity shows 2-4 dB improvement compared to equal power allocation case. Calculation of LDPC thresholds with this power allocation in OFDM subcarriers may look like a new challenge. We assumed that the input to each OFDM subcarriers is binary with BPSK modulation and hence the signal power in each subcarrier is constant. We formulated the density evolution algorithm (which gives us the LDPC thresholds) with these assumptions. So, how we calculate the LDPC thresholds when the signal power in each subcarriers is different is not very clear. But, this difficulty can be overcome by a simple manipulation. We can write $X_i = \sqrt{p_i P} S_i$, where where S_i is unit power input. The the normalized powers p_i are constrained by $\frac{1}{N_c}\sum_{i=1}^{N_c} p_i \leq 1$ so that overall power constraint is satisfied. Now, we can subsume this $\sqrt{p_i P}$ factor to the H_i and rewrite (10) as

$$Y_i = H_i' S_i + W_i, \qquad (11)$$

Now LDPC thresholds can be calculated with the same algorithm.

VII. Results

We give a few results of the thresholds estimated using the proposed algorithm.

A. Threshold Evaluation

We consider a 2-tap channel,

$$\{h[i]\} = \{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}.$$

We use this channel to compare results with those published in [3].

(L, R)	**Rate**	**ISI threshold** [3]		**OFDM threshold**	
		σ^*	SNR*	σ^*	SNR*
(3, 4)	0.250	1.196	−1.555	1.175	−1.413
(3, 5)	0.400	0.945	0.491	0.897	0.9376
(3, 6)	0.500	0.822	1.703	0.751	2.478
(3, 10)	0.700	0.631	3.999	0.479	6.3754
(3, 15)	0.800	0.547	5.240	0.3217	9.840

TABLE I

Regular LDPC code threshold for ISI & OFDM

Rate	**Equal Power**		**Mercury/Waterfilling**	
	σ^*	SNR*	σ^*	SNR*
0.10	1.713	−4.677	2.485	−7.906
0.30	1.176	−1.408	1.493	−3.484
0.60	0.666	3.511	0.737	2.653
0.80	0.365	8.757	0.472	6.515

TABLE II

Irregular LDPC code threshold for OFDM .

Table I gives the thresholds obtained for different rate regular LDPC codes. It gives the degree distribution of the LDPC code, its design rate, the threshold values obtained[3] for OFDM and also lists the thresholds obtained for ISI channel using the *BCJR* algorithm in [3].

Table II gives the thresholds for irregular LDPC codes. These codes are optimized for AWGN channel.

[3]The threshold values are scaled by $\sqrt{2}$ to compare with the corresponding values of ISI thresholds, wherein the σ values correspond to the variance of the real part of the noise.

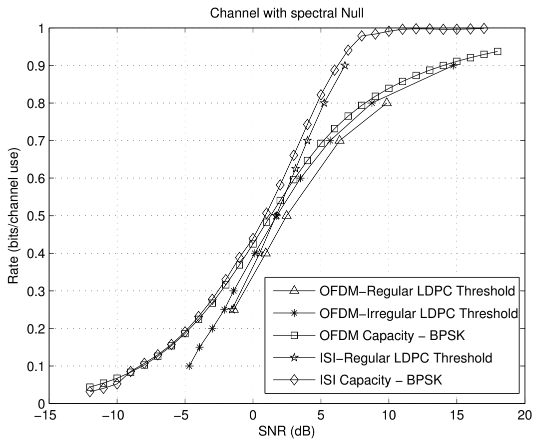

Fig. 1. OFDM and ISI Thresholds

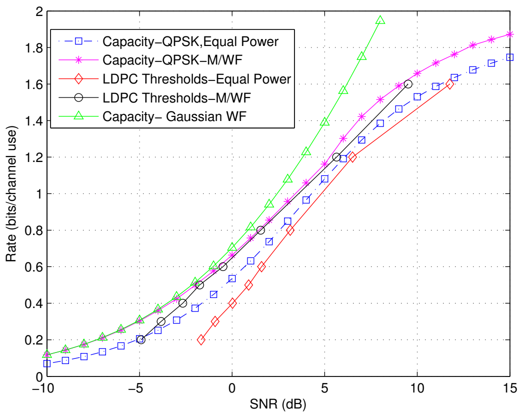

Fig. 2. LDPC Thresholds for OFDM with Mercury/Waterfilling power allocation.

Figure 1 summarizes the results. It gives the LDPC thresholds (for regular and irregular codes) over OFDM and LDPC thresholds over an ISI channel. For comparison, we also plot OFDM capacity and ISI capacity. We see that irregular codes gives an improvement of 1dB over the regular codes. At higher rates LDPC threshold (for irregular codes) are very close to the theoretical limit, the OFDM capacity. LDPC thresholds for ISI channel is also very close to ISI channel capacity at higher rates. In both case, the gap between the capacity and LDPC thresholds increases as the rate decreases. We can also compare OFDM-LDPC thresholds and ISI-LDPC thresholds. We see that for lower rate OFDM-LDPC thresholds are better than that of ISI-LDPC thresholds. But as the rate increases, ISI-LDPC thresholds get better and at higher rates ISI-LDPC threshold are very much superior to OFDM-LDPC thresholds. One can observe a fundamental reason behind this. In this region, ISI capacity is much better than the OFDM capacity, which can be seen clearly from the figure. OFDM-LDPC threshold can get maximum upto the OFDM channel capacity and at higher rate they do so. However the ISI capacity is much better than these values and ISI-LDPC codes perform close to those. So, ISI-LDPC thresholds are fundamentally better than OFDM-thresholds in this region.

We apply Mercury/Waterfilling power allocation policy to OFDM and calculate the LDPC threshold in this case. Table II gives the OFDM-LDPC thresholds for irregular codes. OFDM capacity with Mercury/Waterfilling power allocation and LDPC thresholds in this case are plotted in the figure 2 along with equal power case (We have considered QPSK modulation here). LDPC thresholds show a 2-4 dB improvement. More interestingly, LDPC thresholds in this case are very close to the capacity even at moderate rates.

VIII. Conclusion

In this work, we have proposed a rigorous density evolution (without Gaussian approximation) to analyze the performance of LDPC codes over an OFDM system. We proved the existence of LDPC threshold in an OFDM system and calculated the LDPC threshold for various regular and irregular codes. We have shown that, for higher rates the irregular LDPC thresholds are very close to OFDM capacity. Then, we used Mercury/Waterfilling power allocation for OFDM subcarriers and calculated the LDPC thresholds in this case. In this case, we demonstrated that, irregular LDPC threshold are close to capacity even at moderate rates.

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