MMSE DFE for MIMO DFT-spread OFDMA

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***Abstract*— In this work, we propose a Decision feedback equalizer (DFE) for Multiple Input Multiple Output (MIMO) DFT-spread OFDMA. This DFE can handle Multiple users, multiple antennas at the transmitter and receiver and the inter-symbol interference in the received symbols of the DFT-spread OFDMA. The conventional method used in DFT-spread OFDMA is linear Frequency domain equalization (FDE). The DFE, with a feed forward (FF) filter operating in frequency domain and a feed back (FB) filter operating in the time domain can perform better than FDE, in channels with severe ISI. The block-wise circular convolution of the channel impulse response makes it difficult to initialize the DFE. We use an LE to get a temporary set of decisions and use them in initializing the DFE.**

I. INTRODUCTION

DFT spread-Orthogonal Frequency Division Multiple Access(DFT spread OFDMA)/Single carrier Frequency Domain Multiple Access (SC-FDMA) has drawn great attention as an attractive alternative to Orthogonal Frequency Division Multiple Access (OFDMA) , especially in the uplink communications where lower PAPR greatly benefits the mobile terminal in terms of transmit power efficiency. An added advantage of DFT spread-OFDMA is that coding, while desirable, is not necessary for combating frequency selectivity, as it is in nonadaptive OFDM. It is currently a working assumption for uplink multiple access scheme in 3GPP Long Term Evolution (LTE) and it is proposed for the IEEE 802.16m Wireless MAN standard. The performance of conventional FDE is not enough for channels with severe ISI. An obvious alternative is the DFE, which makes use of previous decisions in attempting to estimate the current symbol with a symbol-by-symbol detector. Any tailing ISI caused by a previous symbol is reconstructed and then subtracted.

A DFE for SISO single carrier modulation schemes is proposed by Benvenuto *et al* [4], where the initialization of DFE is achieved by a PN extension. In this paper, we propose a DFE for MIMO DFT-spread OFDMA, where the data from a user occupies a localized/distributed subset of subcarriers in the frequency band. We need to operate on this block which, in time domain, is circularly convolved with the channel. The basic MIMO DFE structure is formulated in Naofal *et al* [1]. The multiple inputs can be transmitted from multiple users, where each is equipped with a single antenna or a single user (e.g., a base station) equipped with multiple antennas or the combinations of both. [1] deals with the equalization where the channel convolution is linear. The MIMO DFE operates to remove both the intersymbol and inter-antenna interference. The Widely-linear MIMO DFE is discussed in Mattera *et al* [2]. In our work, we reformulate the DFE for the DFT spread-OFDMA scenario, where the channel convolution is circular.

The major difficulty here, is that the ISI in the initial symbols are from the last symbols of the data block, and they are not detected yet. To overcome this, A Linear equaliser is first run and then use the decisions from it, to initialize the DFE. The Feed forward section is operating in the frequency domain, which reduces the complexity by avoiding the circular convolution.

A Frequency domain block iterative DFE for single carrier modulation, which has a similar performance to the time-domain DFE is proppsed in Benvenuto *et al* [3].

II. SYSTEM DESCRIPTION

A. Equalization for DFT spread OFDMA

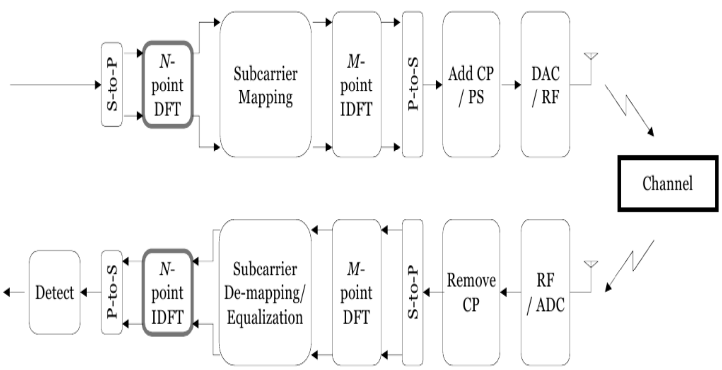

Fig. 1. DFT spread OFDMA/SC-FDMA system

Frequency domain linear equalization in a Single Carrier (SC) system is simply the frequency domain analog of what is done by a conventional linear time domain equalizer. For channels with severe delay spread, frequency domain equalization is computationally simpler than corresponding time domain equalization, because it is performed on a block of data at a time, and the operations on this block involve an efficient FFT operation and a simple channel inversion operation.

DFE gives better performance for frequency-selective radio channels than linear equalization. In conventional DFE equalizers, symbol-by-symbol data decisions are made, filtered, and immediately fed back to remove their interference effect from subsequently detected symbols. Because of the delay inherent in the block FFT signal processing, this immediate filtered decision feed-back cannot be done in a frequency domain DFE, which uses frequency domain filtering of the fed-back signal. A hybrid time-frequency domain DFE approach, which avoids the above mentioned feedback delay problem, would be to use frequency domain filtering only for the forward filter part of the DFE, and conventional transversal filtering for the feedback part. The transversal feedback filter is relatively simple in any case, since it performs multiplications only on data symbols, and it could be made as short or long as required for adequate performance. Fig 2 illustrates such a hybrid time-frequency domain DFE topology. A similar DFE structure for SC systems is discussed in Falconer *et al* [5]. The N DFT output symbols (after the subcarrier demapping) are multiplied by the complex-valued forward equalizer coefficients which compensate for the frequency selective channel variations of amplitude and phase with frequency. An IFFT is applied to the equalized symbols, and the resulting time-domain sequence is passed to a data symbol decision device or, in the case of a DFE, the estimated ISI due to previously detected symbols is computed using a feedback filter, and subtracted off, symbol by symbol.

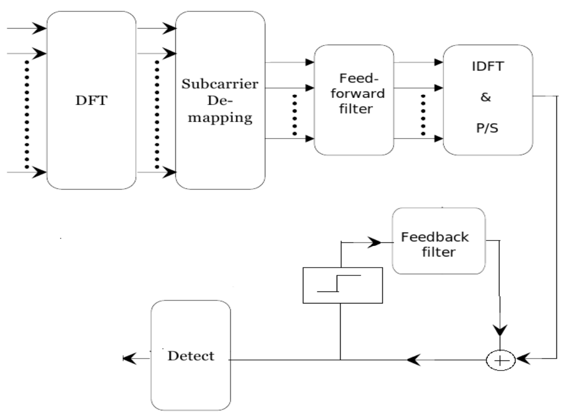

Fig. 2. The hybrid time frequency DFE for DFT spread OFDMA

B. MIMO DFT precoded-OFDMA Model

The output signal of the IDFT block in (2), in the absence of any equalisation, can be represented as

$$y_k = \sum_{m=0}^{v} h_m x_{(k-m)_N} + n_k, \quad (1)$$

for $k = 0, 1...N-1$. where x_ks are the transmitted symbols and $h_m, m = 0, 1, ..v$ are the $v+1$ channel coefficients. Now consider a MIMO DFT precoded OFDMA system with n_i input streams and n_o output streams. The Samples at the j^{th} output can be written as

$$y_k^j = \sum_{i=0}^{n_i} \sum_{m=0}^{v^{(i,j)}} h_m^{(i,j)} x^i_{(k-m)_N} + n_k^j, \quad (2)$$

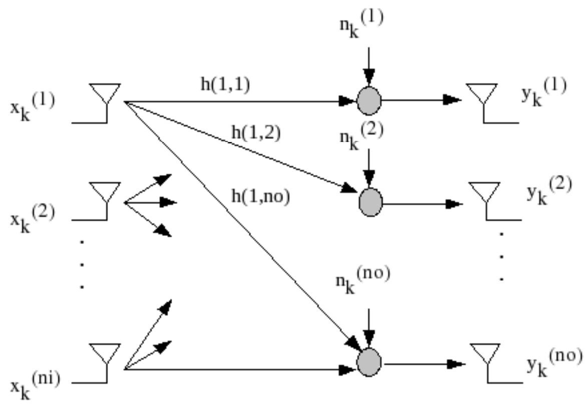

Fig. 3. The MIMO Model

where x_k^i is the sample at the i^{th} input stream. $h_m^{(i,j)}$ is the channel impulse response between input and output whose memory is $v^{(i,j)}$. $()_N$ denotes the modulo N operation.

Here, we are trying to find out an MMSE DFE solution in the time domain (ie, after IDFT block).

Arranging all y_k^j s in one column vector as $\mathbf{y}_k$ and all x_k^j s in one column vector as $\mathbf{x}_k$, it can be written as

$$\mathbf{y}_k = \sum_{m=0}^{v} \mathbf{H}_m \mathbf{x}_{(k-m)_N} + \mathbf{n}_k, \quad (3)$$

where, $\mathbf{H}_m$s are the $n_o \times n_i$ channel matrices. $v = max(v^{(i,j)})$.

Now consider a block of N_f output symbols

$$\mathbf{y}_{k:k-N_f+1} = \mathbf{H}\mathbf{x}_{k:k-N_f+1} + \mathbf{n}_{k:k-N_f+1} \quad (4)$$

where,

$$\begin{aligned} \mathbf{y}_{k:k-N_f+1} &= \begin{bmatrix} \mathbf{y}_{(k)_N} & \mathbf{y}_{(k-1)_N} & . & . & \mathbf{y}_{(k-N_f+1)_N} \end{bmatrix}^T, \\ \mathbf{x}_{k:k-N_f+1} &= \begin{bmatrix} \mathbf{x}_{(k)_N} & \mathbf{x}_{(k-1)_N} & . & . & \mathbf{x}_{(k-N_f+1)_N} \end{bmatrix}^T, \\ \mathbf{x}_{k:k-N_f+1} &= \begin{bmatrix} \mathbf{x}_{(k)_N} & \mathbf{x}_{(k-1)_N} & . & . & \mathbf{x}_{(k-N_f+1)_N} \end{bmatrix}^T, \end{aligned}$$

and

$$\mathbf{H} = \begin{bmatrix} \mathbf{H_0} & \mathbf{H_1} & . & . & \mathbf{H_v} & \mathbf{0} & . & \mathbf{0} \\ \mathbf{0} & \mathbf{H_0} & \mathbf{H_1} & . & . & \mathbf{H_v} & \mathbf{0} & . \\ . & . & . & . & . & . & . & . \\ \mathbf{H_1} & \mathbf{H_2} & . & . & \mathbf{0} & \mathbf{0} & . & \mathbf{H_0} \end{bmatrix}$$

It can be written more compactly as

$$\mathbf{Y}_k = \mathbf{H}\mathbf{X}_k + \mathbf{N}_k \quad (5)$$

H is a block circulant matrix. Now define $\mathbf{R_{xx}}$ as the Covariance matrix of $\mathbf{X}_k$, $\mathbf{R_{nn}}$ as the Covariance matrix of $\mathbf{N}_k$ and $\mathbf{R_{yy}}$ as the Covariance matrix of $\mathbf{Y}_k$. From (5)

$$\mathbf{R_{yy}} = \mathbf{H}\mathbf{R_{xx}}\mathbf{H^H} + \mathbf{R_{nn}}, \quad (6)$$

and the cross-covariance matrix between input and output

$$\mathbf{R_{xy}} = \mathbf{R_{xx}}\mathbf{H^H}. \quad (7)$$

Now, consider the linear equalizer formulation. The estimate of $\mathbf{x}_{k-\Delta}$,

$$\widehat{\mathbf{x}}_{k-\Delta} = \mathbf{W_{LE}^H}\mathbf{Y}_k, \tag{8}$$

where Δ is the decision delay. The orthogonality condition is

$$\mathbb{E}[(\mathbf{x}_{k-\Delta} - \mathbf{W_{LE}}^H\mathbf{Y}_k)\mathbf{Y}_k^H] = \mathbf{0}. \tag{9}$$

$\mathbf{x}_{k-\Delta}$ can be obtained from $\mathbf{X}_k$:

$$\mathbf{x}_{k-\Delta} = \mathbf{M}\mathbf{X}_k, \tag{10}$$

where $\mathbf{M} = [\mathbf{0}_{n_i \times n_i \Delta}\mathbf{I}_{n_i}\mathbf{0}_{n_i \times n_i(N_f-\Delta-1)}]$. Using the orthogonality condition, the LE can expressed as

$$\mathbf{W_{LE}^H} = \mathbf{M}\mathbf{R_{xy}}\mathbf{R_{yy}^{-1}} \tag{11}$$

Now, perform the equalisation (ie, convolution), only for the last few symbols, in the time domain (ie, after the IDFT in (2)). The symbol decisions from this LE will be later used in initializing the DFE.

III. MIMO MMSE DFE

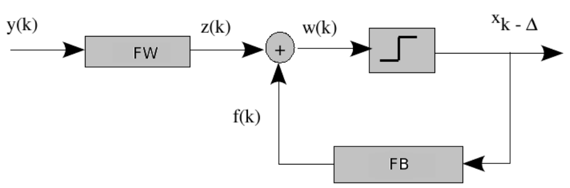

Fig. 4. The Decision Feedback equalizer

The Feed forward filter matrix is given by

$$\mathbf{W^H} = \begin{bmatrix} \mathbf{W_0^H} & \mathbf{W_1^H} & \mathbf{W_2^H} & . & . & \mathbf{W_{N_f-1}^H} \end{bmatrix}, \tag{12}$$

where each $\mathbf{W_i}$ is an $n_o \times n_i$ matrix. $N_f < N$ is the number of Feed forward filter taps. The feedback filter is

$$\tilde{\mathbf{B}} = [\mathbf{I}_{n_i}\mathbf{0}_{n_i \times n_i . N_b}] - \mathbf{B_t^H}, \tag{13}$$

where N_b is the number of feedback filter taps and

$$\mathbf{B_t^H} = \begin{bmatrix} \mathbf{B_0^H} & \mathbf{B_1^H} & \mathbf{B_2^H} & . & . & \mathbf{B_{N_b}^H} \end{bmatrix}. \tag{14}$$

The output signal of the feed forward filter is

$$\mathbf{z}(k) = \mathbf{W}^H\mathbf{Y}_k. \tag{15}$$

Assuming correct decisions (Remember that, the decisions from the LE are available), the signal which is fed back can be written as

$$\mathbf{f}(k) = \tilde{\mathbf{B}} \begin{bmatrix} \mathbf{x}_{(k-\Delta)_N} \\ \mathbf{x}_{(k-\Delta-1)_N} \\ . \\ . \\ \mathbf{x}_{(k-\Delta-N_b)_N} \end{bmatrix} = [\mathbf{0}_{n_i \times n_i \Delta}\tilde{\mathbf{B}}]\mathbf{X}_k. \tag{16}$$

Thus

$$\Delta + N_b = N_f - 1; N_b = N_f - \Delta - 1. \tag{17}$$

The signal after the removal of ISI is

$$\mathbf{w}(k) = \mathbf{z}(k) + \mathbf{f}(k) = \mathbf{W}^H\mathbf{Y}_k + [\mathbf{0}_{n_i \times n_i \Delta}\tilde{\mathbf{B}}]\mathbf{X}_k. \tag{18}$$

The error vector is

$$\mathbf{E}_k = \mathbf{x}_{k-\Delta} - \mathbf{w}(k). \tag{19}$$

It can be written as

$$\mathbf{E}_k = [\mathbf{0}_{n_i \times n_i \Delta}\mathbf{I}_{n_i}\mathbf{0}_{n_i \times n_i N_b}]\mathbf{X}_k - \mathbf{W}^H\mathbf{Y}_k - [\mathbf{0}_{n_i \times n_i \Delta}\tilde{\mathbf{B}}]\mathbf{X}_k. \tag{20}$$

Assuming that a user has access to the previous decisions of all other users, $\mathbf{B_0}$ is kept equal to $\mathbf{I}_{n_i}$, so that

$$\tilde{\mathbf{B}} = -\begin{bmatrix} \mathbf{0} & \mathbf{B_1^H} & \mathbf{B_2^H} & . & . & \mathbf{B_{N_b}^H} \end{bmatrix}. \tag{21}$$

Now define $\mathbf{B^H} = [\mathbf{0}_{n_i \times n_i \Delta}\mathbf{B_t^H}]$. The error vector can now be expressed as

$$\mathbf{E}_k = \mathbf{B}^H\mathbf{X}_k - \mathbf{W}^H\mathbf{Y}_k. \tag{22}$$

Now, observe that

$$\mathbf{B^H}\mathbf{\Phi} = \mathbf{C^H} \tag{23}$$

where $\mathbf{\Phi} = \begin{bmatrix} \mathbf{I}_{n_i(\Delta+1)} \\ \mathbf{0}_{n_i N_b \times n_i(\Delta+1)} \end{bmatrix}$ and $\mathbf{C^H} = [\mathbf{0}_{n_i \times n_i \Delta}\mathbf{I}_{n_i}]$

The error covariance matrix is $\mathbf{R_{ee}} = \mathbb{E}[\mathbf{E}_k\mathbf{E}_k^H]$

$$= \mathbf{B^H}\mathbf{R_{xx}}\mathbf{B} + \mathbf{W^H}\mathbf{R_{yy}}\mathbf{W} - \mathbf{B^H}\mathbf{R_{xy}}\mathbf{W} - \mathbf{W^H}\mathbf{R_{yx}}\mathbf{B}. \tag{24}$$

From the orthogonality condition

$$\mathbb{E}[\mathbf{E}_k\mathbf{Y}_k^H] = \mathbf{0} \tag{25}$$

The feed-forward filter can be expressed as

$$\mathbf{W_{opt}^H} = \mathbf{B_{opt}^H}\mathbf{R_{xy}}\mathbf{Ryy}^{-1}. \tag{26}$$

Using (26) the error covariance matrix can be written as

$$\mathbf{R_{ee}} = \mathbf{B^H}\mathbf{R}^{-1}\mathbf{B}, \tag{27}$$

where

$$\mathbf{R}^{-1} = \mathbf{R_{xx}} - \mathbf{R_{xy}}\mathbf{R_{yy}^{-1}}\mathbf{R_{yx}} \tag{28}$$

Using the matrix inversion Lemma, We can see that

$$\mathbf{R} = \mathbf{R_{xx}^{-1}} + \mathbf{H^H}\mathbf{R_{nn}^{-1}}\mathbf{H}. \tag{29}$$

The Optimum Feedback filter is obtained by minimizing the trace of $\mathbf{R_{ee}}$, with the constraint (23). This gives the solution for the feed-back filter

$$\mathbf{B_{opt}} = \mathbf{R}\mathbf{\Phi}(\mathbf{\Phi^H}\mathbf{R}\mathbf{\Phi})^{-1}\mathbf{C}. \tag{30}$$

And the $\mathbf{R_{ee}}$ corresponding to this MMSE DFE is

$$\mathbf{R_{ee,min}} = \mathbf{C^H}(\mathbf{\Phi^H}\mathbf{R}\mathbf{\Phi})^{-1}\mathbf{C}. \tag{31}$$

Now make the tap length of $\mathbf{W_{opt}^H}$ equal to N, by zero padding. Take a block-wise FFT of this time domain FF filter and perform the feed-forward filtering in the frequency domain (ie, before the IDFT in fig(2) , and the feed-back filter in the time domain (ie, after the IDFT).

Note that there is no need to calculate the LE coefficients in (11) independently. It can be extracted during the FF filter computation in (26) , thereby reducing the complexity.

IV. RESULTS

Observe that the uncoded BER of the MIMO DFE in fading channel is almost insensitive to the decision delay Δ. The BER performances of a '1 Tranmit 2 Receive antenna (1 TX 2 RX)' and a '2 TX 2 RX Spatial Multiplexing' DFT precoded OFDMA are studied. Simulations are done for blocks of 64 QPSK symbols circularly convolved with a 5 tap iid Gaussian channel. There is a 1 dB gain for '1 TX 2 RX' and a 3 dB gain for '2 TX 2 RX' in SNR for DFE, compared to LE at BEP 10^{-3}.

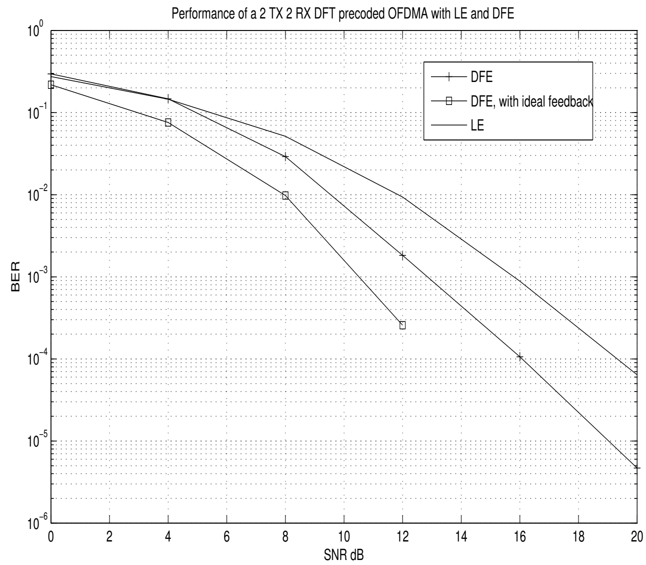

Fig. 5. Uncoded BER comparison of the DFE and LE for 2 TX 2 RX

The Block error rate comparisons between DFE and LE for a 1TX 2RX DFT-precoded OFDMA system in fading channel is shown in Fig 7. QPSK modulation with rate 5/6 convolutional Turbo code with block size 12 bytes is used and ideal channel estimation is assumed. The FFT Size used is 1024 and the frequency Comb-size allocated for a user is 16. The simulation is run for a Doppler spread of 7 Hz. There is a 1 dB gain in SNR for DFE, compared to LE at 1 percent Block error rate.

V. CONCLUSION

In this work, we have proposed a hybrid time-frequency domain DFE for MIMO DFT spread OFDMA. From the Uncoded BER performance in fading channel, it is clear that the DFE achieves more diversity gain compared to that of LE. The advantage of DFE is evident from the BLER curves with convolutional Turbo code too. One of the facts for further study is the complexity reduction of the Feed-forward and Feed back filter calculations based on the circulant property of the channel matrix.

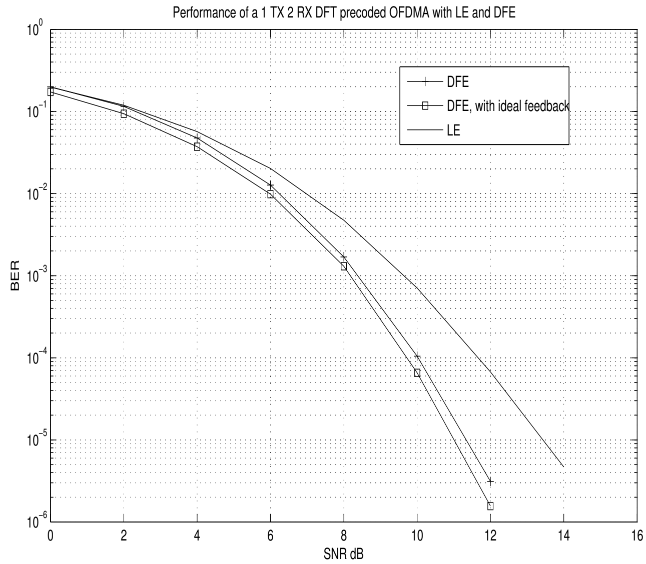

Fig. 6. Uncoded BER comparison of the DFE and LE for 1 TX 2 RX

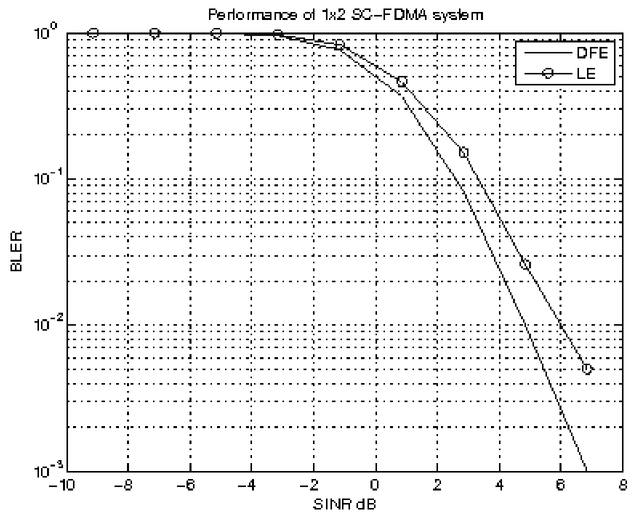

Fig. 7. BLER comparison of the DFE and LE for 1TX 2RX

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